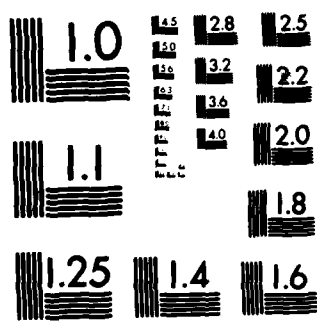


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TECHNICAL REPORT ARAED-TR-86022

**SKY SCREEN DATA REDUCTION TO OBTAIN MUZZLE VELOCITY  
AND LINEAR DRAG COEFFICIENT**

CHIU H. NG

**AUGUST 1986**



US ARMY  
ARMAMENT  
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**U. S. ARMY ARMAMENT RESEARCH, DEVELOPMENT AND ENGINEERING CENTER**

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## INTRODUCTION

For most of the tank projectile test firings, sky screens are used to obtain the flight time as a function of range. These range data can be used to extract muzzle velocity, velocity decay, and drag coefficient. The most common technique to extract muzzle velocity is to use the difference method to obtain the secant

velocities (i.e.,  $V_{x_i} = \frac{x_{i+1} - x_i}{t_{i+1} - t_i}$ ). The first few secant velocities are then

fitted with a straight line or a parabola to obtain the muzzle velocity. There are two disadvantages with this type of data reduction technique: (1) the difference method magnifies the error in obtaining the secant velocities, (2) the velocity profile is neither a straight line nor a parabola. To remove these disadvantages, a closed form solution to the equations of motion is used to fit the flight time from the sky screens. A closed form solution is possible when the drag coefficient is assumed to be a linear function of velocity. Therefore, the drag coefficient can also be obtained by fitting the closed form solution to the sky screen data.

## ANALYSIS

The equation of motion for a flat fire antitank projectile is

$$M\dot{V} = -\frac{1}{2}\rho V^2 S C_D \quad (1)$$

where

$M$  = Projectile mass  
 $V, \dot{V}$  = Velocity and time rate of change of velocity  
 $\rho$  = Air density  
 $S$  = Reference area  
 $C_D$  = Drag coefficient

When the drag coefficient is a linear function of velocity, having a form of  $C_D = C_{D_a}(1 + bV)$ , equation 1 becomes

$$M\dot{V} = -\frac{1}{2}\rho V^2 C_{D_a}(1 + bV)$$

or

$$\dot{V} = -AV^2(1 + bV) \quad (2)$$

where  $A$  is a constant and is defined as  $\frac{1}{2M}\rho S C_{D_a}$ , and  $b$  is a slope value for the linear drag expression. In most cases,  $b$  has a negative value.

The closed form solution for velocity as a function of range (x) can be obtained by integrating equation 2 with  $V = \frac{dx}{dt}$ . The solution is

$$V = V_c(1 + bV)e^{-Ax}$$

or

$$V = \frac{V_c}{e^{Ax} - bV_c} \quad (3)$$

where  $V_c \equiv \frac{V_0}{1 + bV_0}$  with the initial condition of  $V = V_0$  at  $x = 0$ .

Furthermore, the closed form solution of  $t(x)$  can be obtained from differential equation 3. It has the solution of

$$t = t_0 + \frac{1}{AV_c}(e^{Ax} - 1) - bx \quad (4)$$

with the initial condition of  $t = t_0$  at  $x = 0$ .

Equation 4 expresses the projectile flight time as a function of range which is in the form to fit the sky screen data.

To obtain a best fit in the sense of minimizing the squares of the deviations, a residual is defined

$$R_f = \frac{1}{2} \sum_{i=1}^N (t_{ic} - t_{ie})^2 \quad (5)$$

where  $t_{ic}$  is the time of flight calculated from equation 4 and  $t_{ie}$  is the time of flight obtained from the sky screen for the projectile to reach the  $i^{th}$  sky screen, and  $N$  is the total number of sky screens used for that test.

To minimize the residual, equation 5 is differentiated with respect to the four variables ( $t_0$ ,  $A$ ,  $V_0$ , and  $b$ ) individually, and then set to zero. They are

$$\begin{aligned} \frac{\partial R_f}{\partial t_0} &= \sum_{i=1}^N (t_{ic} - t_{ie}) \frac{\partial t_{ic}}{\partial t_0} = 0 \\ \frac{\partial R_f}{\partial A} &= \sum_{i=1}^N (t_{ic} - t_{ie}) \frac{\partial t_{ic}}{\partial A} = 0 \\ \frac{\partial R_f}{\partial V_0} &= \sum_{i=1}^N (t_{ic} - t_{ie}) \frac{\partial t_{ic}}{\partial V_0} = 0 \\ \frac{\partial R_f}{\partial b} &= \sum_{i=1}^N (t_{ic} - t_{ie}) \frac{\partial t_{ic}}{\partial b} = 0 \end{aligned} \quad (6)$$

The following partial derivatives of  $t_{ic}$  with respect to the four variables can be obtained by differentiating equation 4

$$\begin{aligned}\frac{\partial t_{ic}}{\partial t_o} &= 1 \\ \frac{\partial t_{ic}}{\partial A} &= \frac{1}{A^2 V_c} [1 + e^{Ax_i} (Ax_i - 1)] \\ \frac{\partial t_{ic}}{\partial V_o} &= -\frac{1}{AV_o^2} (e^{Ax_i} - 1) \\ \frac{\partial t_{ic}}{\partial b} &= (e^{Ax_i} - 1) - x_i\end{aligned}\quad (7)$$

With such a complex expression of the four variables in the set of equation 6, the easier approach in solving the four unknown variables is by using the iterative method. To obtain the corrections for the variables from the  $j^{th}$  iteration to the  $j + 1^{st}$  iteration, the  $t_{ic}$  is expanded in a Taylor's Series and the series is truncated after the first order terms. Therefore

$$t_{ic}(j+1) = t_{ic}(j) + \frac{\partial t_{ic}}{\partial t_o} \bigg|_j (\Delta t_o) + \frac{\partial t_{ic}}{\partial A} \bigg|_j (\Delta A) + \frac{\partial t_{ic}}{\partial V_o} \bigg|_j (\Delta V_o) + \frac{\partial t_{ic}}{\partial b} \bigg|_j (\Delta b) \quad (8)$$

Equation 8 is substituted into equation 6 for calculating the  $j + 1^{st}$  corrections. For simplicity, the subscript  $j$  and  $j + 1$  will be omitted in the following expressions:

$$\begin{aligned}\frac{\partial R_o}{\partial t_o} = 0 &= \sum_{i=1}^N [(t_{ic} - t_{ic}) \frac{\partial t_{ic}}{\partial t_o} + (\frac{\partial t_{ic}}{\partial t_o})^2 \Delta t_o + \frac{\partial t_{ic}}{\partial t_o} \frac{\partial t_{ic}}{\partial A} \Delta A + \frac{\partial t_{ic}}{\partial t_o} \frac{\partial t_{ic}}{\partial V_o} \Delta V_o + \frac{\partial t_{ic}}{\partial t_o} \frac{\partial t_{ic}}{\partial b} \Delta b] \\ \frac{\partial R_o}{\partial A} = 0 &= \sum_{i=1}^N [(t_{ic} - t_{ic}) \frac{\partial t_{ic}}{\partial A} + \frac{\partial t_{ic}}{\partial t_o} \frac{\partial t_{ic}}{\partial A} \Delta t_o + (\frac{\partial t_{ic}}{\partial A})^2 \Delta A + \frac{\partial t_{ic}}{\partial A} \frac{\partial t_{ic}}{\partial V_o} \Delta V_o + \frac{\partial t_{ic}}{\partial A} \frac{\partial t_{ic}}{\partial b} \Delta b] \\ \frac{\partial R_o}{\partial V_o} = 0 &= \sum_{i=1}^N [(t_{ic} - t_{ic}) \frac{\partial t_{ic}}{\partial V_o} + \frac{\partial t_{ic}}{\partial t_o} \frac{\partial t_{ic}}{\partial V_o} \Delta t_o + \frac{\partial t_{ic}}{\partial A} \frac{\partial t_{ic}}{\partial V_o} \Delta A + (\frac{\partial t_{ic}}{\partial V_o})^2 \Delta V_o + \frac{\partial t_{ic}}{\partial V_o} \frac{\partial t_{ic}}{\partial b} \Delta b] \\ \frac{\partial R_o}{\partial b} = 0 &= \sum_{i=1}^N [(t_{ic} - t_{ic}) \frac{\partial t_{ic}}{\partial b} + \frac{\partial t_{ic}}{\partial t_o} \frac{\partial t_{ic}}{\partial b} \Delta t_o + \frac{\partial t_{ic}}{\partial A} \frac{\partial t_{ic}}{\partial b} \Delta A + \frac{\partial t_{ic}}{\partial V_o} \frac{\partial t_{ic}}{\partial b} \Delta V_o + (\frac{\partial t_{ic}}{\partial b})^2 \Delta b]\end{aligned}\quad (9)$$

The set of equation 9 can be written in matrix notation, which is in the form of:  $M \cdot \Delta B = C$



where

$$M = \begin{pmatrix} \sum_{i=1}^N \left( \frac{\partial t_i}{\partial t_0} \right)^2 & \sum_{i=1}^N \frac{\partial t_i}{\partial t_0} \frac{\partial t_i}{\partial A} & \sum_{i=1}^N \frac{\partial t_i}{\partial t_0} \frac{\partial t_i}{\partial V_0} & \sum_{i=1}^N \frac{\partial t_i}{\partial t_0} \frac{\partial t_i}{\partial b} \\ \sum_{i=1}^N \frac{\partial t_i}{\partial t_0} \frac{\partial t_i}{\partial A} & \sum_{i=1}^N \left( \frac{\partial t_i}{\partial A} \right)^2 & \sum_{i=1}^N \frac{\partial t_i}{\partial A} \frac{\partial t_i}{\partial V_0} & \sum_{i=1}^N \frac{\partial t_i}{\partial A} \frac{\partial t_i}{\partial b} \\ \sum_{i=1}^N \frac{\partial t_i}{\partial t_0} \frac{\partial t_i}{\partial V_0} & \sum_{i=1}^N \frac{\partial t_i}{\partial A} \frac{\partial t_i}{\partial V_0} & \sum_{i=1}^N \left( \frac{\partial t_i}{\partial V_0} \right)^2 & \sum_{i=1}^N \frac{\partial t_i}{\partial V_0} \frac{\partial t_i}{\partial b} \\ \sum_{i=1}^N \frac{\partial t_i}{\partial t_0} \frac{\partial t_i}{\partial b} & \sum_{i=1}^N \frac{\partial t_i}{\partial A} \frac{\partial t_i}{\partial b} & \sum_{i=1}^N \frac{\partial t_i}{\partial V_0} \frac{\partial t_i}{\partial b} & \sum_{i=1}^N \left( \frac{\partial t_i}{\partial b} \right)^2 \end{pmatrix}$$

$$\Delta B = \begin{pmatrix} \Delta t_0 \\ \Delta A \\ \Delta V_0 \\ \Delta b \end{pmatrix} \quad \text{and } C = \begin{pmatrix} \sum_{i=1}^N (t_{ie} - t_{ic}) \frac{\partial t_i}{\partial t_0} \\ \sum_{i=1}^N (t_{ie} - t_{ic}) \frac{\partial t_i}{\partial A} \\ \sum_{i=1}^N (t_{ie} - t_{ic}) \frac{\partial t_i}{\partial V_0} \\ \sum_{i=1}^N (t_{ie} - t_{ic}) \frac{\partial t_i}{\partial b} \end{pmatrix} \quad (10)$$

The subscript c for the partial derivatives has been omitted. There is no ambiguity because the partial derivatives can only be computed from the closed form solution. The corrections are obtained by inverting matrix M, that is

$$\Delta B = M^{-1}C \quad (11)$$

The iteration will be terminated when a prescribed degree of convergence or number of iterations is reached.

## DISCUSSION

The computer program listing for this sky screen data reduction technique is shown in appendix A. The sky screen data for round number 63 for the 105-mm APFSDS projectile M833 fired on 20 April 1984 at Aberdeen Proving Ground was chosen as a test case for this data reduction technique. The computer printout for this test case is given in appendix B. The results show that it only took four iterations to achieve the desired convergence even though the initial guessed muzzle velocity and drag coefficient were different by as much as 35% from their best-fit values. Therefore, for reasonably well-behaved sky screen data, this iterative method converges very rapidly.

To investigate the deviation between the path the projectile travelled and the ground distance due to the trajectory curvature, the deviations were calculated for ground ranges of 2,500 meters and 3,000 meters for the 105-mm HEAT projectile M456 and the 105-mm APFSDS projectile M833. The super-elevation needed to reach 2,500 meters for the M456 projectile is 0.84 degree and to reach 3,000 meters for the M833 projectile is 0.41 degree. This deviation is approximately 0.025 meters for the M833 projectile and 0.07 meters for the M456 projectile at their respective ranges. These deviations are within the accuracy of the sky screen data, therefore, the trajectory curvature can be neglected in the data reduction.

### CONCLUSIONS

The only limitation of this sky screen data reduction technique is that the drag coefficient has to be a linear function of velocity. From results of some test cases, it indicates that this technique still rendered excellent results, even with the drag coefficient deviating slightly from the linearity requirement.

# SYMBOLS

A	Constant, defined as $\frac{1}{2M} \rho S C_{D_a}$ , per meter
b	Slope value in drag coefficient expression, per meter per second
$C_D$	Drag coefficient
$C_{D_a}$	Constant in drag coefficient expression
M	Projectile mass, kg
$\rho$	Air density, kg/meter <sup>3</sup>
S	Projectile reference area, meter <sup>2</sup>
t	Time of flight, seconds
$t_0$	Time at the gun muzzle, seconds
V	Projectile velocity, meters per second
$V_0$	Muzzle velocity, meters per second
$V_c$	Defined as $\frac{V_0}{1 + bV_0}$ , meters per second
x	Range, meters
( $\dot{\phantom{x}}$ )	Time rate of change, $\frac{d}{dt}(\phantom{x})$

## Subscript

c	Values computed from the closed form solution
e	Values from range data
i	Values pertaining to the i <sup>th</sup> sky screen
j	Values pertaining to the j <sup>th</sup> iteration

**APPENDIX A**  
**COMPUTER PROGRAM LISTING**

```

100      PROGRAM MUZFIT(INPUT,TAPE5=INPUT,OUTPUT,TAPE6=OUTPUT)
110      DIMENSION X(16),TE(16),TC(16),TD(16),WXX(5),C(5,6),D(5)
120      DIMENSION AA(4),M(16),XX(16),TITLE(8),FF(4)
130      DIMENSION R(7),V(7)
140      DATA XX/56.81,74.81,92.81,110.8,128.81,239.76,453.22,779.93,
150      +982.1,1251.18,1479.0,1741.45,1984.35,2475.85,2733.21,2983.21/
160      DATA AA/3H10=,3HA=,3HV0=,3HB= /
170      DATA FF/.010,.00005,30.,.00005/
180      DATA R/0.0,500.0,1000.0,1500.0,2000.0,2500.0,3000.0/
190      700 FORMAT(8A10)
200      701 FORMAT(16I1)
210      702 FORMAT(8F10.2)
220      10 READ(5,700) TITLE
230      IF(EOF(5) .NE. 0.0) STOP
240      READ(5,702) (TE(I),I=1,16)
250      READ(5,702) (D(I),I=1,4)
260      READ(5,701) M
270      L=0
280      DO 50 I=1,16
290      IF(M(I) .EQ. 0) GO TO 50
300      L=L+1
310      X(L)=XX(I)
320      TE(L)=TE(I)
330      50 CONTINUE
340      NT=L
350      800 FORMAT(1H1)
360      WRITE(6,800)
370      600 FORMAT(1H0/10X,8A10)
380      WRITE(6,600) TITLE
390      601 FORMAT(/10X,*NUMBER OF SKY SCREENS = *,I2)
400      WRITE(6,601) NT
410      WRITE(6,801)
420      801 FORMAT(/10X,*RANGE DATA-METERS*/)
430      WRITE(6,602) (X(I),I=1,NT)
440      602 FORMAT( 10X,10F10.3)
450      WRITE(6,802)
460      802 FORMAT(/10X,*TIME DATA-SECONDS*/)
470      WRITE(6,603) (TE(I),I=1,NT)
480      603 FORMAT( 10X,10F10.6)
490      WRITE(6,803)
500      803 FORMAT(///10X,*INITIAL GUESSES---*)
510      WRITE(6,604) (D(I),I=1,4)
520      604 FORMAT( /10X,F12.6 ,F12.7,F12.2,F12.6)
530      NR=0
540      E=0.0
550      805 FORMAT(/4X,13HITERATION NO.,6X,9HRESIDUALS,7X,14HPROBABLE ERROR
560      +//10X,43HCORRECTIONS OF CONSTANTS FOR EACH ITERATION)
570      WRITE(6,805)

```

```

580      100 SR=0.0
590      DO 130 J=1,5
600      WXX(J)=0.0
610      DO 130 I=1,4
620      C(I,J)=0.0
630      130 CONTINUE
640      TO=D(1)
650      A=D(2)
660      VO=D(3)
670      B=D(4)
680      VC=VO/(1.0+B*VO)
690      AVC=A*VC
700      AAVC=A*AVC
710      VC2=VC**2
720      VO2=VO**2
730      DO 150 I=1,NT
740      AX=A*X(I)
750      EAX=EXP(AX)
760      EAX1=EAX-1.0
770      PTA=(-EAX1+EAX*AX)/AAVC
780      PTV=-EAX1/(A*VO2)
790      PTB=(EAX1-AX)/A
800      TC(I)=TO+EAX1/AVC-B*X(I)
810      TD(I)=TE(I)-TC(I)
820      SR=SR+TD(I)**2
830      C(1,1)=C(1,1)+1.0
840      C(2,1)=C(2,1)+PTA
850      C(3,1)=C(3,1)+PTV
860      C(4,1)=C(4,1)+PTB
870      C(1,2)=C(2,1)
880      C(2,2)=C(2,2)+PTA**2
890      C(3,2)=C(3,2)+PTA*PTV
900      C(4,2)=C(4,2)+PTA*PTB
910      C(1,3)=C(3,1)
920      C(2,3)=C(3,2)
930      C(3,3)=C(3,3)+PTV**2
940      C(4,3)=C(4,3)+PTV*PTB
950      C(1,4)=C(4,1)
960      C(2,4)=C(4,2)
970      C(3,4)=C(4,3)
980      C(4,4)=C(4,4)+PTB**2
990      C(1,5)=C(1,5)+TD(I)
1000     C(2,5)=C(2,5)+TD(I)*PTA
1010     C(3,5)=C(3,5)+TD(I)*PTV
1020     C(4,5)=C(4,5)+TD(I)*PTB
1030     150 CONTINUE
1040     NR=NR+1
1050     E1=E

```

```

1060      E=0.6745*SQRT(SR/(NT-4))
1070      WRITE(6,605) NR,SR,E
1080      605 FORMAT(/10X,12,2F20.8)
1090      IF (ABS(E1-E)-1.0E-05) 200,200,110
1100      110 IF (NR-15) 120,120,200
1110      120 CALL INV(C,4,5,WXX)
1120          DO 180 I=1,4
1130              F=C(I,5)
1140              IF (ABS(F) .GT. FF(I)) F=SIGN(FF(I),F)
1150              D(I)=D(I)+F
1160      180 CONTINUE
1170      WRITE(6,606) (C(I,5),I=1,4)
1180      606 FORMAT(/10X,5E18.6)
1190      GO TO 100
1200      200 CONTINUE
1210      CALL INV(C,4,4,WXX)
1220          DO 210 I=1,4
1230      210 WXX(I)=SQRT(ABS(C(I,I)))*E
1240      806 FORMAT(/22X,15HBEST-FIT VALUES,4X,14HPROBABLE ERROR)
1250      WRITE(6,806)
1260          DO 230 I=1,4
1270      230 WRITE(6,607) AA(I),D(I),WXX(I)
1280      607 FORMAT(/10X,A3,5X,2E18.6)
1290      804 FORMAT(/13X,13HRANGE-METERS ,13HTIME-OBS-SEC ,13HTIME-CAL-SEC
1300          *12HTIME-DIF-SEC//)
1310      608 FORMAT(10X,F13.2,F13.6,F13.6,F13.6)
1320      WRITE(6,804)
1330          DO 240 I=1,NT
1340      WRITE(6,608) X(I),TE(I),TC(I),TD(I)
1350      240 CONTINUE
1360      V1=D(3)+D(1)*D(2)*D(3)**2*(1.0+D(3)*D(4))
1370      WRITE(6,609) V1
1380      609 FORMAT(/10X,*MUZZLE VELOCITY AT TIME EQUAL TO ZERO IS *,F10.2)
1390      V(1)=D(3)
1400      VC=D(3)/(1.0+D(4)*D(3))
1410          DO 250 I=2,7
1420      VAX=VC*EXP(-D(2)*(I-1)*500.0)
1430      V(I)=VAX/(1.0-D(4)*VAX)
1440      250 CONTINUE
1450      WRITE(6,610) R
1460      WRITE(6,611) V
1470      610 FORMAT(/5X,15HRANGE(METERS) =,7F12.1)
1480      611 FORMAT(5X,15HVELOCITY(MPS) =,7F12.1)
1490      GO TO 10
1500      END

```

```

1510     SUBROUTINE INV(C,NC,NCS1,WXX)
1520     DIMENSION C(5,6),WXX(5),PIVOT(2),CC(5,10)
1530     NCT=NC*2
1540     NCP1=NC+1
1550     DO 10 I=1,NC
1560     DO 10 J=1,NC
1570     10 CC(I,J)=C(I,J)
1580     DO 20 I=1,NC
1590     DO 20 J=NCP1,NCT
1600     20 CC(I,J)=0.0
1610     DO 30 I=1,NC
1620     30 CC(I,NC+I)=1.0
1630     DO 205 I=1,NC
1640     PIVOT(1)=CC(I,I)
1650     DO 200 K=1,NC
1660     PIVOT(2)=CC(K,I)
1670     120 IF(K-I) 135,130,140
1680     130 DO 150 J=1,NCT
1690     IF(PIVOT(1)) 134,210,134
1700     134 CC(K,J)=CC(I,J)/PIVOT(1)
1710     150 CONTINUE
1720     GO TO 200
1730     135 DO 160 J=1,NCT
1740     IF(PIVOT(1)) 136,160,136
1750     136 CC(K,J)=CC(K,J)-CC(I,J)*PIVOT(2)/PIVOT(1)
1760     160 CONTINUE
1770     GO TO 200
1780     140 DO 170 J=1,NCT
1790     IF(PIVOT(2)) 145,170,145
1800     145 CC(K,J)=CC(K,J)/PIVOT(2)-CC(I,J)
1810     170 CONTINUE
1820     200 CONTINUE
1830     205 CONTINUE
1840     GO TO 250
1850     210 WRITE(6,600)
1860     600 FORMAT(//10X,*DET IS EQUAL ZERO*)
1870     250 DO 300 I=1,NC
1880     DO 300 J=1,NC
1890     300 C(I,J)=CC(I,J+NC)
1900     NCS=NCS1
1910     350 IF(NCS-NC) 500,500,400
1920     400 DO 420 I=1,NC
1930     WXX(I)=C(I,NCS)
1940     420 C(I,NCS)=0.0
1950     DO 450 I=1,NC
1960     DO 450 J=1,NC
1970     450 C(I,NCS)=C(I,NCS)+C(I,J)*WXX(J)
1980     NCS=NCS-1
1990     GO TO 350
2000     500 CONTINUE
2010     RETURN
2020     END

```



**APPENDIX B**  
**COMPUTER OUTPUT EXAMPLE**

MR33 RD-63 20 APRIL 1984

NUMBER OF SKY SCREENS = 16

RANGE DATA-METERS

56.810	74.810	92.810	110.800	128.810	239.760	982.100	1251.180
1479.000	1741.450	1984.350	2475.850	2733.210	2983.210		

TIME DATA-SECONDS

.019120	.031357	.043615	.055884	.068176	.144037	.517793	.659608
1.013619	1.202379	1.379403	1.743115	1.935780	2.126484		

INITIAL GUESSES--

-.01000	.0001000	1420.00	-.000450
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ITERATION NO.	RESIDUALS	PROBABLE ERROR
CORRECTIONS OF CONSTANTS FOR EACH ITERATION		

1	.03476655	.03630547	
	-.06241E-02	.333420E-04	.481413E+02
			-.881337E-04

2	.00562211	.01459963	
	-.333491E-06	.219809E-05	.195644E+02
			-.131498E-04

3	.00000128	.00022051	
	-.014364E-07	.270118E-06	.266375E+00
			.237251E-07

4	.00000108	.00020221	
	-.105127E-07	.311720E-08	-.105189E-03
			-.460922E-08

	BEST-FIT VALUES	PROBABLE ERROR
T0=	-.196246E-01	.109928E-03
A=	.135813E-03	.209134E-04
V0=	.146983E+04	.971986E+00
B=	-.513131E-03	.314583E-04

# RANGE-METERS TIME-OBS-SEC TIME-CAL-SEC TIME-DIF-SEC

56.81	.019129	.019063	.000357
74.81	.031357	.031336	.000020
92.81	.043615	.043617	-.000002
110.80	.055884	.055898	-.000314
128.81	.068176	.068201	-.000025
139.76	.144037	.144156	-.000119
452.22	.291479	.291105	.000374
779.93	.517793	.518159	-.000366
982.10	.659608	.660304	-.000396
1251.18	.850710	.850444	.000266
1479.80	1.013619	1.013203	.000416
1741.45	1.202379	1.202492	-.000113
1984.35	1.379403	1.379445	-.000042
2475.85	1.743115	1.742932	.000183
2733.21	1.935780	1.936293	-.000513
2983.21	2.126484	2.126212	.000272

MUZZLE VELOCITY AT TIME EQUAL TO ZERO IS 1468.42

RANGE(METERS) =	0.0	500.0	1000.0	1500.0	2000.0	2500.0	3000.0
VELOCITY(MPS) =	1469.8	1444.9	1419.1	1392.5	1365.1	1337.0	1308.1

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